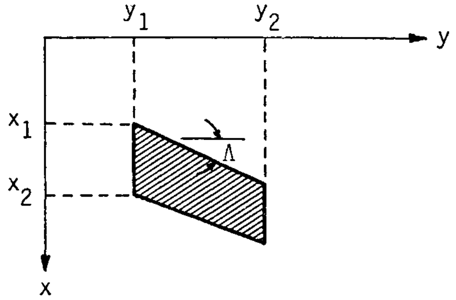

Fig. 1 Parallelogram patch.

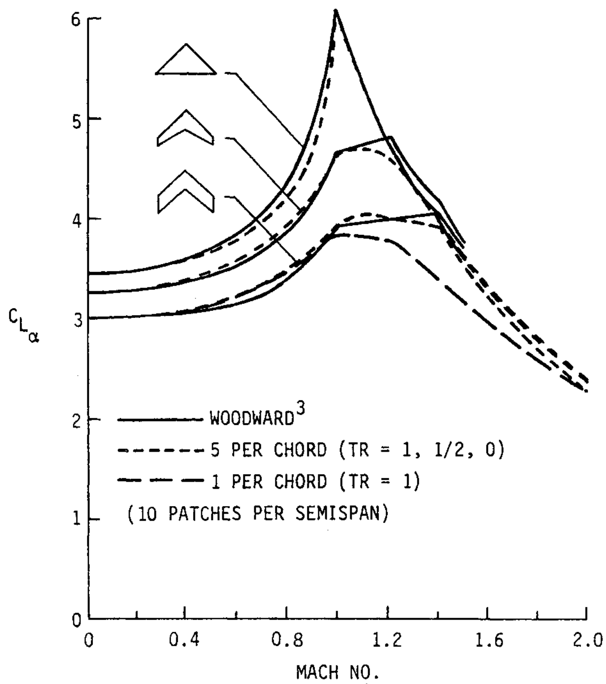

Fig. 2 Lift-curve slopes, aspect ratio 4 wings.

$$I=\int_{y_1}^{y_2}\mathrm{d}y\int_{x_1}^{x_2}\frac{(x-\lambda y_1+\lambda y)\mathrm{d}x}{y^2R(x-\lambda y_1+\lambda y,y)}$$

$$\approx 2\epsilon\sum_{k=1}^{N}\frac{R(x_2+\Delta x_k,y_k)-R(x_1+\Delta x_k,y_k)}{y_k^2-\epsilon^2} \tag{1a}$$

$$\lambda=\tan\Lambda \tag{1b}$$

$$\epsilon=(y_2-y_1)/2N \tag{1c}$$

$$y_k=y_1+(2k-1)\epsilon \tag{1d}$$

$$\Delta x_k=(2k-1)\lambda\epsilon \tag{1e}$$

$$R(x,y)=[x^2+(1-M^2)y^2]^{1/2},\qquad M<1$$

$$=\{\mathrm{Max}[0,\mathrm{Max}^2(0,x)+(1-M^2)y^2]\}^{1/2}, M>1 \tag{1f}$$

Second, the most desirable relative location of the doublet/averaging area is that which yields lifting line results when used subsonically with one patch per chord. When averaging over a parallelogram patch, the preferred doublet location is $1/(e^2+1)=11.92\%$ of patch chord at the spanwise patch center, whereas Ref. 1 used the leading edge. This is discussed in Ref. 2 for the analogous downwash-point method. [Reference 2 also used unsteady/steady kernal ratios for supersonic solutions, as in Eqs. (19a) and (19b) of Ref. 1, but did not recognize the extra term in Eq. (19c), Ref. 1.] Figure 2 illustrates the use of this doublet location and of Eq. (1) with $N=5$, using the planforms of Ref. 3. Patch centers of pressure are not dictated by the doublet location used in forming the influence matrix, but may be assumed to be at 25% chord for $M<1$, at 50% chord for supersonic patch leading edges, and interpolated for in-between Mach numbers.

References

[1]Ueda, T. and Dowell, E. H., "Doublet-Point Method for Supersonic Unsteady Lifting Surfaces," *AIAA Journal*, Vol. 22, Feb. 1984, pp. 179-186.

[2]Roger, K. L., "Airplane Math Modeling Methods for Active Control Design," *AGARD CP-228*, April 1977, pp. 4.1-4.11.

[3]Woodward, F. A., "Analysis and Design of Wing-Body Combinations at Subsonic and Supersonic Speeds," *Journal of Aircraft*, Vol. 5, Nov.-Dec. 1968, pp. 528-534.

Reply by Authors to K. L. Roger

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WE wish to thank K. L. Roger for his interest and remarks concerning our paper.[1] His first Comment suggests replacement of our analytical results for the integration of an $x/(y^2R)$ function by a numerical quadrature. Let us consider the unswept case for simplicity; then we can find that Eq. (1a) of the Comment corresponds to neglecting the second term $\sin^{-1}(\beta y/x)$ of Eq. (6c) in Ref. 1. The approximation may be allowable provided that contributions of the inverse sine term are small in comparison with those from the first term, $R/(\beta y)$, which is the case for a small $|\beta y|$ or ϵ. Since it is easy to compute the inverse sine function by using modern computers, we do not think that the replacement of our analytical integration by Eq. (1a) is always efficient. Although analytical integration is also possible for swept and/or tapered regions, we use rectangular geometry for the upwash averaging area, believing that the effects of the shape deformation are of the same order as of the descretization error.

The second Comment by K. L. Roger is on the location of the area. He referred to the method described in Ref. 2, which seems to be based on the constant pressure panel method[3] using the velocity potential. Unfortunately, we could not discuss the method in detail since in Ref. 2 there is no detailed formulation but only a brief description. The location recommended in the Comment could be worthy of a trial. The optimal locations for the doublet points and upwash averaging area are uncertain. As stated in Ref. 1, they are determined by trial-and-error considering the trading-off of the undulation of the solution and the smearing of pressures at Mach lines from break-off points in the leading edge. Our results, however, show satisfactory pressure distributions and good convergence as the number of elements increases.

References

[1]Ueda, T. and Dowell, E. H., "Doublet-Point Method for Supersonic Unsteady Lifting Surfaces," *AIAA Journal*, Vol. 22, Feb. 1984, pp. 179-186.

[2]Roger, K. L., "Airplane Math Modeling Methods for Active Control Design," AGARD CP-228, April 1977, p. 4.1-4.11.

[3]Woodward, F. A., "Analysis and Design of Wing-Body Combinations at Subsonic and Supersonic Speeds," *Journal of Aircraft*, Vol. 4, Nov.-Dec. 1968, pp. 528-534.

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